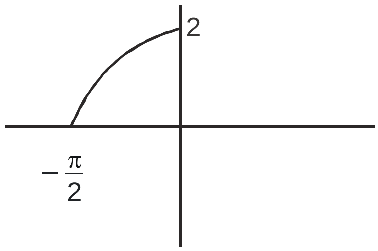
EXERCISE – II — HINTS & SOLUTIONS

Sol.1 A,B

$f(x) = x + \sin x$



$f'(x) = 1 + \cos x$

$-1 \le \cos x \le 1$

$0 \le 1 + \cos x \le 2$

Increasing

Sol.2 B,C

$f(x) = 2\ell n\,(x-2) - x^2 + 4x + 1$

$$f'(x) = \frac{2}{x-2} - 2x + 4$$

$$= 2\left[\frac{1}{(x-2)} - (x-2)\right]$$

$x = 2, 3, 1$ critical points

$\uparrow$in $x \in (-\infty, 1) \cup (2, 3)$

Sol.3 A,D

$f(x) = 2x + \cos^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$

$$f'(x) = 2 - \frac{1}{1+x^2} + \frac{1}{(\sqrt{1+x^2} - x)}\left(\frac{2x}{2\sqrt{1+x^2}} - 1\right)$$

$$= 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}$$

$$= \frac{2(1+x^2) - 1 - \sqrt{1+x^2}}{(1+x^2)}$$

$$= \frac{(1+x^2) + x^2 - \sqrt{1+x^2}}{(1+x^2)} > 0$$

$f(x) \uparrow$ is $(-\infty, \infty)$

Sol.4 B,C

$$g(x) = 2f\left(\frac{x}{2}\right) + f(1-x)$$

$$g'(x) = f'\left(\frac{x}{2}\right) - f'(1-x) = 0 \Rightarrow \frac{x}{2} = 1 - x$$

$$\Rightarrow \boxed{x = \frac{2}{3}} \text{ critical point.}$$

$$g''(x) = \frac{1}{2} f''\left(\frac{x}{2}\right) + f''(1-x)$$

$f''(x) < 0$ for $0 \le x \le 1$

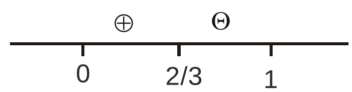
$f''(x/2) < 0$

$$0 \le \frac{x}{2} \le \frac{1}{2}$$

$0 \le 1 - x \le 1 \qquad \Rightarrow f''(1-x) < 0$

$g''(x) < 0 \Rightarrow g'(x)$ is decreasing function

(4), (1) because $\downarrow$function



$$x \in \left[0, \frac{2}{3}\right] \uparrow \text{ and } \left[\frac{2}{3}, 1\right] \downarrow$$

Sol.5 B,C

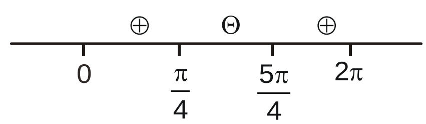
$f(x) = \sin x + \cos x$

$f'(x) = \cos x - \cos x = 0$

$\tan x = 1$

$$x = n\pi + \frac{\pi}{4}$$

$$n = 0,\ x = \frac{\pi}{4}$$



$$n = 1,\ x = \frac{5x}{4}$$

$$\downarrow \text{ is } \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$$

$$\uparrow \text{ is } \left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$$

Sol.6 A,B,C

$$f(x) = \tan^{-1} x - \frac{1}{2}\,\ell nx$$

$$f'(x) = \frac{1}{1+x^2} - \frac{1}{2x} = \frac{2x-(1+x^2)}{2x(1+x^2)} = \frac{-(x-1)^2}{2x(1+x^2)}$$

Θ

0

f(x) is $\downarrow$ for x > 0

Greatest value will be at $x = \frac{1}{\sqrt{3}}$

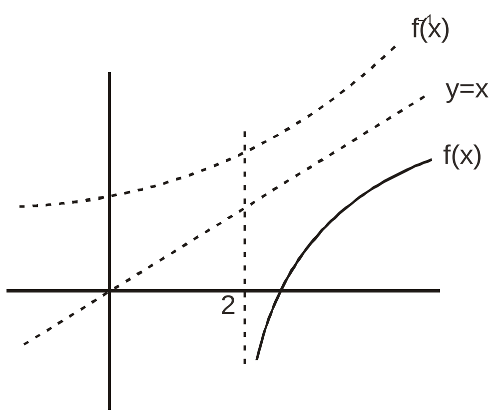
$$f\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\frac{1}{\sqrt{3}} - \frac{1}{2}\,\ell n\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{1}{4}\,\ell n3$$

least value will be at $x = \sqrt{3}$

$$f(v) = \tan^{-1}\sqrt{3} - \frac{1}{2}\,\ell n\sqrt{3} = \frac{\pi}{3} - \frac{1}{4}\ell n3$$

Sol.7 A,C,D

$$f(x) = \log(x-2) - \frac{1}{x}$$



$$f'(x) = \frac{1}{x-2} + \frac{1}{x^2} = \frac{x^2+x-2}{x^2(x-2)}$$

$$f'(x) = \frac{(x+2)(x-1)}{x^2(x-2)}$$

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–2 0 1 2

$f^{-1}(x)$ is monotonic increasing.

Sol.8 A,D

(A) f(0) = 0

f(0) = 0

$f(\pi/2) = e^{\pi/2}$

Rolle's theorem Not applicable

(b) f(–1) = f (3/2) = 0

(c) f(x) = sin |x|

differentiation is $[\pi, 2\pi]$

$f(x) = 0 = f(2\pi)$

$$D = \sin\frac{1}{x}$$

Sol.9 A,B

$f(x) = x^{m/n}$

Let assume m = 2, n = 3

$f(x) = x^{2/3}$

$$f'(x) = \frac{2}{3}\,x^{-1/3}$$

Θ $\oplus$

0

f(x) is $\downarrow$ is $x \in (-\infty, 0)$

in $x \in (0, \infty)$

Sol.10 A,D

h(x) = f(x) g(x)

$\Rightarrow$ f(x) g'(x) + g(x) f'(x) > 0

+ve + – –

h is increasing

h(x) = fog(x) $\Rightarrow$ h'(x) = f'(g(x). g'(x) < 0

h is decreasing

Sol.11 A,B

$y = 2x^2 - \ell n\,|x|$

$$y' = 4x - \frac{1}{x} = \frac{4x^2-1}{x} = \frac{(2x-1)(2x+1)}{x}$$

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$-\frac{1}{2}$ 0 $\frac{1}{2}$

$$I_1 : x \in \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$$

$$I_2 : x \in \left(-\infty, -\frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$$

Sol.12 **A,D**

$\phi'(x) = 3f^2 f' - 6ff' + 4f' + 5 + 3\cos x - 4\sin x$

$= f'(x)\,[3f^2 - 6f + 4]$

$\downarrow$

$d < 0$

$\phi'(x)$ increasing as f increasing

Sol.13 **A,C**

$\phi(x) = f(x) + f(2a - x)$

$\phi'(x) = f'(x) - f'(2a - x)$

$x = 2a - x \Rightarrow \boxed{x = a}$ critical points

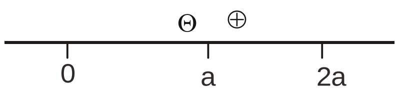
$\phi''(x) = f''(x) + f''(2a - x)$

$f''(x) > 0$ for $0 \le x \le 2a$

$f''(2a - x) > 0 \quad 0 \le 2a - x \le 2a$

$\phi'(x) > 0$

$\phi'(x)$ is increasing



$x \in (0, a) \downarrow$

$x \in (a, 2a) \uparrow$

Sol.14 **A,B,C,D**

$f'(x) = 16x^3(3\ln x - 1) = 0$

$x = e^{1/3}$ is mining

$f''(x) = 16(9x^2 \ln x) = 0$

upward $(1, \infty)$

downward $(0, 1)$

$x = 1 \Rightarrow f(1) = -7$

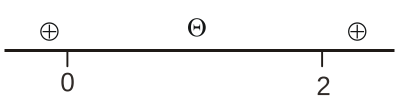
Sol.15 **A,B**

$f(x) = 3x^4 + 4x^3 - 12x^2 - 7$

$f'(x) = 12x^3 + 12x^2 - 24x$

$= 12x(x^2 + x - 2)$

$= 12x(x + 2)(x - 1)$



$\downarrow$ is $(-2, 0) \cup (1, \infty)$

$\uparrow x \in (-\infty, 0) \cup (2, \infty)$

Sol.16 **C,D**

$f(x) = x + 1\ \dfrac{1}{x-1}$

$f'(x) = 1 - \dfrac{1}{(x-1)^2} = 0$

x = 0, 2 are critical point

increasing $x \in [0, 1)$ u $(1, 2]$

increasing $x \in (-\infty\ , 0)$ U $[2, \infty)$

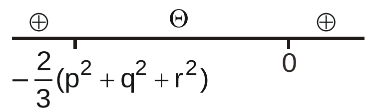
Sol.17 **A,B**

$f(x) = x^2(x + p^2 + q^2 + r^2)$

By using determinant prop.

$f'(x) = 2x(x + p^2 + q^2 + r^2) + x^2$

$= x[3x + 2p^2 + 2q^2 + 2r^2]$



$\uparrow x < -\dfrac{2}{3}(p^2 + q^2 + r^2) \cup x > 0$

$\downarrow x \in \left(-\dfrac{2}{3}(p^2 + q^2 + r^2), 0\right)$

Sol.18 **A,C**

(A) $f(z) = \tan^{-1} z$

$f'(z) = \dfrac{f(y) - f(x)}{y - x}$

$\left|\dfrac{1}{1+z^2}\right| = \left|\dfrac{\tan^{-1}y - \tan^{-1}x}{y - x}\right|$

$\left|\dfrac{\tan^{-1}y - \tan^{-1}x}{y - x}\right| \le 1$

Aliter

$|\tan^{-1}x - \tan^{-1}y| \le |y - x|$

This will be is $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

But RHS can infinite so always true.

(B) $f(z) = \sin z$

$f'(z) = \cos z$

$\dfrac{f(y) - f(x)}{y - x} = \cos z \Rightarrow \left|\dfrac{\sin y - \sin x}{y - x}\right| = |\cos z| \le 1$

$|\sin y - \sin x| \le |y - x|$

Aliter

$|\sin y - \sin x| \le |y - x|$

$\downarrow$

LHS as $x \to \infty$,

a finite quantity between – 1 to 1

RbS will be infinitely

which is always two.